Set-valued risk measures – Set-valued risk measure processes

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Résumé

In their seminal paper [1], Artzner et al. adopt an axiomatic approach to characterize economically coherent risk measures. In their setting, a financial position is identified with its *final net worth* and modeled by a real-valued random variable $X \in L^{\infty}(\Omega, \mathcal{F}, \mathbb{P})$. The investor defines among the set of all possible outcomes a subset $\mathcal{A}$ of *acceptable positions* regarded as risk-free. Then the risk measure $\rho(X)$ of X corresponds to the *'extra' capital requirement* that has to be invested in some *secure* instrument so that the resulting position is acceptable : $X + \rho(X) \in \mathcal{A}$. The authors define a set of properties to be satisfied by ρ, which can de translated in terms of properties to be satisfied by the acceptance set $\mathcal{A}$, in order to make the risk measure *economically sensible.*

The concept of coherent risk measures has been extended to *set-valued risk measures* by Jouni et al [2]. Their approach stems form the fact that in realistic situations, investors have access to different markets and form multi-assets portfolios ; in the presence of *market frictions*, a financial positions cannot be merely described by a real-valued random net-worth. Hence, in their setting, they describe a position by an $\mathbb{R}^d$-valued random variable, $X \in L^{\infty}_d = L^{\infty}_d(\Omega, \mathcal{F}, \mathbb{P})$ and they assume that L^{∞}_d is endowed with a partial preorder relation $\succeq$ which accounts for the market's frictions. A risk measure R is then defined as a *set-valued* mapping which associates to $X \in L^{\infty}_d$ the set of deterministic risk-free portfolios $x \in \mathbb{R}^d$ such that : $X + x$ is acceptable.

This talk is based on a joint work with Emmanuel Denis [3]. Its main purpose is to extend the notion of coherent *set-valued* risk measures in a dynamic setting. Indeed, as time goes by, it seems natural to update the risk of a given position according to new information. Given a final time horizon T, at each date $t \in [0, T]$, we shall adjust the set-valued risk-measure of $X \in L^{\infty}_d$, and consider a set-valued risk-measure process $\{R_t(X),\ t \in [0, T]\}$. We propose to define a coherent risk-measure process among random-vectors of L^0_d which are ordered according to a *random* cone G_T. We discuss the issue of *time consistency* and provide two examples : (i). a *distribution based* set-valued risk measure process which relies on a convenient definition of *quantiles for vector-valued* random variables, (ii). a risk measure process based on d-dimensional BSDE's.

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Références

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[2] Jouini E., Meddeb M. and Touzi N. Vector valued coherent risk measures. *Finance and Stochastics* 8 (2004), 4, 531-552.

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